APEX 2011 Workshop

Ring Transfer and Coupling Matrix at IP

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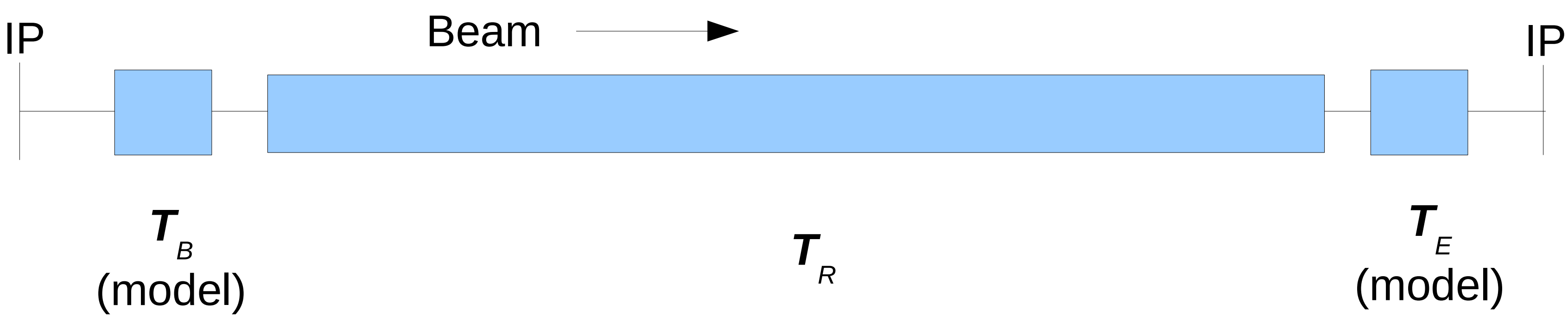
Ring Transfer and Coupling Matrix

- Description of the method
- Simulation results
- First attempts
- Conclusions

Description of Method

Extending the method of measuring β* by varying the two quadrupoles about the IP.



The ring transfer matrix: $$T = T_E T_R T_B = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

T can be described with 10 parameters: $[\mu_x, \alpha_x, \beta_x, \mu_y, \alpha_y, \beta_y, a, b, c, d]$

[A] Vary a quadrupole in T_B: $$T_\Delta = T_E T_R T_{B+\Delta} = T_E T_R T_B (T_B^{-1} T_{B+\Delta}) = T (T_B^{-1} T_{B+\Delta})$$

[B] Vary a quadrupole in T_E: $$T_\Delta = T_{E+\Delta} T_R T_B = (T_{E+\Delta} T_E^{-1}) T_E T_R T_B = (T_{E+\Delta} T_E^{-1}) T$$

Description of Method, cont...

The eigen-tunes from the transfer matrix:

$$Q_{\pm} = Tune_{\pm}(\boldsymbol{T}) = \frac{1}{2\pi}\arccos\left(\frac{1}{2}(Tr(\boldsymbol{A})+Tr(\boldsymbol{D})) \pm \sqrt{\frac{1}{4}(Tr(\boldsymbol{A})-Tr(\boldsymbol{D}))^2 + det(\bar{\boldsymbol{B}}+\boldsymbol{C})}\right)$$

The ΔQ_{min} from the transfer matrix:

$$\Delta Q_{min} = DtuneMin(\boldsymbol{T}) = \frac{\sqrt{det(\bar{\boldsymbol{B}}+\boldsymbol{C})}}{\pi[\sin(2\pi Q_{+})+\sin(2\pi Q_{-})]}$$

Minimum case using 4 quadrupoles – 15 equations with 10 unknowns:

$$\mu_x = 2\pi Q_{+} \qquad \mu_y = 2\pi Q_{-} \qquad \Delta Q_{min} = DtuneMin(\boldsymbol{T}) \qquad \mu_x > \mu_y$$

$$Q_{\pm}^{(1)} = Tune_{\pm}(\boldsymbol{T}_{\Delta_1}) \qquad \Delta Q_{min}^{(1)} = DtuneMin(\boldsymbol{T}_{\Delta_1}) \qquad Q_{\pm}^{(2)} = Tune_{\pm}(\boldsymbol{T}_{\Delta_2}) \qquad \Delta Q_{min}^{(2)} = DtuneMin(\boldsymbol{T}_{\Delta_2})$$

$$Q_{\pm}^{(3)} = Tune_{\pm}(\boldsymbol{T}_{\Delta_3}) \qquad \Delta Q_{min}^{(3)} = DtuneMin(\boldsymbol{T}_{\Delta_3}) \qquad Q_{\pm}^{(4)} = Tune_{\pm}(\boldsymbol{T}_{\Delta_4}) \qquad \Delta Q_{min}^{(4)} = DtuneMin(\boldsymbol{T}_{\Delta_4})$$

Simulations

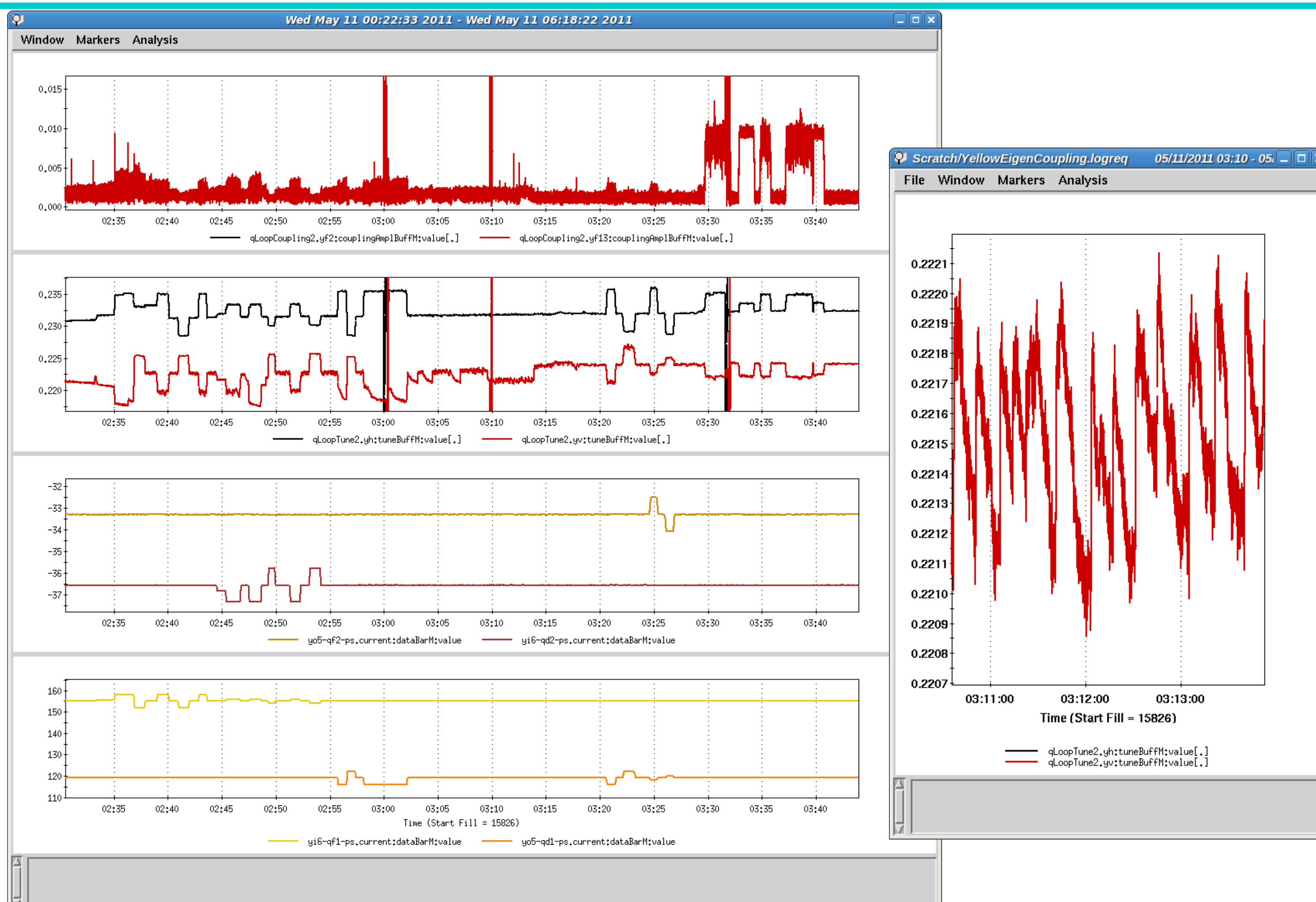
IBS-suppression optics with rolls in the triplets

Model	ALFX	BETX	ALFY	BETY
Case #1	-0.1883	0.7655	0.8647	0.6219
Case #2	-0.2964	1.2880	0.4687	1.2437

	Quadrupole Errors				Case #1				Case #2			
Row	**Q2I**	**Q1I**	**Q1O**	**Q2O**	**ALFX**	**BETX**	**ALFY**	**BETY**	**ALFX**	**BETX**	**ALFY**	**BETY**
0	0%	0%	0%	0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1	-1%	0%	0%	0%	1.17%	0.02%	6.42%	5.20%	2.09%	0.30%	1.88%	0.69%
2	1%	0%	0%	0%	1.20%	0.02%	6.24%	5.44%	2.12%	0.29%	1.93%	0.72%
3	0%	-1%	0%	0%	2.25%	0.11%	6.64%	5.37%	1.35%	0.17%	2.37%	0.84%
4	0%	1%	0%	0%	2.21%	0.11%	6.41%	5.59%	1.36%	0.16%	2.39%	0.88%
5	0%	0%	-1%	0%	4.91%	0.45%	7.72%	7.04%	2.43%	0.35%	2.71%	1.10%
6	0%	0%	1%	0%	5.29%	0.46%	8.56%	7.11%	2.45%	0.37%	2.84%	1.11%
7	0%	0%	0%	-1%	2.78%	0.31%	6.59%	5.97%	3.12%	0.43%	2.14%	0.87%
8	0%	0%	0%	1%	2.97%	0.32%	7.23%	6.05%	3.10%	0.46%	2.24%	0.87%

First Attempts

IP6



Vertical tune was noisier and less reproducible than horizontal tune.

Automated BTF measurements were on.

Conclusions

- Proposed a method to measure the 4x4 transfer matrix including coupling
  - At a minimum, vary 4 quadrupoles
    - Must include rolled/skewed quadrupoles
    - In practice it is better to use 6 quadrupoles
  - May have more than one solution
  - Accurate modeling of the $\boldsymbol{T_B}$ and $\boldsymbol{T_E}$ beam lines are necessary
- Works in simulation
- Needs to show it works on a real machine

Backup Slides

IP8 Data

IP8

Vertical tune was noisier and less reproducible than horizontal tune.

Yellow Inner triplet (yi7) rolls were not measured.

Transfer Matrix

$$\boldsymbol{M}_{x|y}=\begin{bmatrix}\cos(\mu_{x|y})+\alpha_{x|y}\sin(\mu_{x|y}) & \beta_{x|y}\sin(\mu_{x|y})\\ -\frac{1+\alpha_{x|y}^{2}}{\beta_{x|y}}\sin(\mu_{x|y}) & \cos(\mu_{x|y})-\alpha_{x|y}\sin(\mu_{x|y})\end{bmatrix}$$

$$\boldsymbol{U}=\begin{bmatrix}\boldsymbol{M}_{x} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_{y}\end{bmatrix}$$

$$\mu_{x|y}=2\pi Q_{x|y}$$

Transfer Matrix cont...

$$\boldsymbol{G}=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \bar{\boldsymbol{G}}=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \boldsymbol{H}=\frac{1}{\sqrt{1+det(\boldsymbol{G})}}\begin{bmatrix} \boldsymbol{I} & \bar{\boldsymbol{G}} \\ -\boldsymbol{G} & \boldsymbol{I} \end{bmatrix} \quad \boldsymbol{H}^{-1}=\frac{1}{\sqrt{1+det(\boldsymbol{G})}}\begin{bmatrix} \boldsymbol{I} & -\bar{\boldsymbol{G}} \\ \boldsymbol{G} & \boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{T}=\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}=\boldsymbol{H}\,\boldsymbol{U}\,\boldsymbol{H}^{-1}=\boldsymbol{H}\begin{bmatrix} \boldsymbol{M}_x & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_y \end{bmatrix}\boldsymbol{H}^{-1}$$

$$\boldsymbol{A}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_x+\bar{\boldsymbol{G}}\,\boldsymbol{M}_y\,\boldsymbol{G}) \qquad \boldsymbol{B}=\frac{1}{1+det(\boldsymbol{G})}(\bar{\boldsymbol{G}}\,\boldsymbol{M}_y-\boldsymbol{M}_x\,\bar{\boldsymbol{G}})$$

$$\boldsymbol{C}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_y\,\boldsymbol{G}-\boldsymbol{G}\,\boldsymbol{M}_x) \qquad \boldsymbol{D}=\frac{1}{1+det(\boldsymbol{G})}(\boldsymbol{M}_y+\boldsymbol{G}\,\boldsymbol{M}_x\,\bar{\boldsymbol{G}})$$